

**Position & Source:** Position vector  $\vec{r}$ , source vector  $\vec{r}'$ , separation vector  $\vec{\Delta r} = \vec{r} - \vec{r}'$

**Fundamental Theorems of Vector Calculus:**

$$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \vec{dl} = f(\vec{b}) - f(\vec{a}) \quad \int \nabla \cdot \vec{A} \, d\tau = \oint \vec{A} \cdot \vec{da} \quad \int (\nabla \times \vec{A}) \cdot \vec{da} = \oint \vec{A} \cdot \vec{dl}$$

**Cartesian Coordinates:**  $\vec{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z}$   $d\tau = dx \, dy \, dz$

$$\nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{x} + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{y} + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{z} \quad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

**Spherical Coordinates:**  $x = r \sin\theta \cos\phi$ ,  $y = r \sin\theta \sin\phi$ ,  $z = r \cos\theta$

$$\vec{dl} = dr\hat{r} + r \, d\theta\hat{\theta} + r \sin\theta \, d\phi\hat{\phi} \quad d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \hat{\phi} \quad \nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin\theta} \left( \frac{\partial(\sin\theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) \hat{r} + \frac{1}{r} \left( \frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r} \right) \hat{\theta} + \frac{1}{r} \left( \frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 f}{\partial \phi^2}$$

**Cylindrical Coordinates:**  $x = s \cos\phi$ ,  $y = s \sin\phi$ ,  $z = z$

$$\vec{dl} = ds\hat{s} + s \, d\phi\hat{\phi} + dz\hat{z} \quad d\tau = s \, ds \, d\phi \, dz$$

$$\nabla f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{1}{s} \frac{\partial(s A_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left( \frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{s} + \left( \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left( \frac{\partial(s A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right) \hat{z} \quad \nabla^2 f = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial f}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

**Dirac Delta Function:**  $\delta^3(\vec{\Delta r}) = \frac{1}{4\pi} \nabla \cdot \left( \frac{\vec{\Delta r}}{\Delta r^2} \right)$ ,  $\int \delta^3(\vec{\Delta r}) \, d\tau = 1$  if  $\vec{r}'$  contained in volume

**Irrotational Function (e.g. Electrostatic Field):**  $\nabla \times \vec{E} = 0$   $\vec{E} = -\nabla V$   $\oint \vec{E} \cdot \vec{dl} = 0$

**Solenoidal Function (e.g. Magnetostatic Field):**  $\nabla \cdot \vec{B} = 0$   $\vec{B} = \nabla \times \vec{A}$   $\oint \vec{B} \cdot \vec{da} = 0$

$$\text{Electric Field: } \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{\Delta r^2} \widehat{\Delta r} \, d\tau', \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{\Delta r^2} \widehat{\Delta r} \, da', \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{\Delta r^2} \widehat{\Delta r} \, dl'$$

$$\text{Magnetic Field: } \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} \, d\tau', \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} \, da', \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}') \times \widehat{\Delta r}}{\Delta r^2} \, dl'$$

**Continuity:**  $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$  (=0 in electrostatics/magnetostatics)

**Gauss's Law:**  $\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\epsilon_0$ ,  $\nabla \cdot \vec{E} = \rho/\epsilon_0$ ,  $\oint \vec{D} \cdot d\vec{a} = Q_{f\_enc}$ ,  $\nabla \cdot \vec{D} = \rho_{free}$

**Ampere's Law:**  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ,  $\nabla \times \vec{B} = \mu_0 \vec{J}$ ,  $\oint \vec{H} \cdot d\vec{l} = I_{free\_enc}$ ,  $\nabla \times \vec{H} = \vec{J}_{free}$

**Electric Potential:**  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{\Delta r} d\tau'$ ,  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}')}{\Delta r} da'$ ,  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')}{\Delta r} dl'$   
 $\vec{E} = -\nabla V$ ,  $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l}$ ,  $\nabla^2 V = -\rho/\epsilon_0$

**Magnetic Vector Potential:**  $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\Delta r} d\tau'$ ,  $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}(\vec{r}')}{\Delta r} da'$ ,  $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{\Delta r} dl'$   
 $\vec{B} = \nabla \times \vec{A}$ ,  $\nabla \cdot \vec{A} = 0$  &  $\nabla^2 \vec{A} = -\mu_0 \vec{J}$  (Coulomb gauge)

**Laplace's Equation:**  $\nabla^2 V = 0$  if  $\rho = 0$ ,  $\nabla^2 \vec{A} = 0$  if  $\vec{J} = 0$

**Separation of Variables:**  $\frac{d^2 X}{dx^2} = C_1 X$ ,  $\frac{d^2 Y}{dy^2} = C_2 Y$ ,  $\frac{d^2 Z}{dz^2} = C_3 Z$ ,  $C_1 + C_2 + C_3 = 0$ ,  $V(x,y,z) = X(x)Y(y)Z(z)$

**Separation of Variables (Spherical):**  $V(r, \theta) = \sum_0^\infty \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$

**Multipoles:**  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_0^\infty \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\alpha) \rho(\vec{r}') d\tau'$ ,  $\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \sum_0^\infty \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\alpha) d\vec{l}'$

**Dipoles:**  $\vec{p} = \int \vec{r}' \rho(\vec{r}') d\tau'$ ,  $V_{dip}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$ ,  $\vec{\tau} = \vec{p} \times \vec{E}$ ,  $\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$ ,  $\vec{P} = \vec{p}/\text{volume}$   
 $\vec{m} = I \vec{a}$ ,  $\vec{A}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$ ,  $\vec{\tau} = \vec{m} \times \vec{B}$ ,  $\vec{F} = \nabla(\vec{m} \cdot \vec{B})$ ,  $\vec{M} = \vec{m}/\text{volume}$

**Fields in Matter:**  $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ ,  $\sigma_b = \vec{P} \cdot \hat{n}$ ,  $\rho_b = -\nabla \cdot \vec{P}$   
 $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$ ,  $\vec{K}_b = \vec{M} \times \hat{n}$ ,  $\vec{J}_b = \nabla \times \vec{M}$

**Linear Materials:**  $\vec{P} = \epsilon_0 \chi_e \vec{E}$ ,  $\vec{D} = \epsilon \vec{E} = (1 + \chi_e) \epsilon_0 \vec{E} = \epsilon_r \epsilon_0 \vec{E}$ ,  $\rho_b = -\frac{\chi_e}{1 + \chi_e} \rho_f$   
 $\vec{M} = \chi_m \vec{H}$ ,  $\vec{B} = \mu \vec{H} = (1 + \chi_m) \mu_0 \vec{H}$ ,  $\vec{J}_b = \chi_m \vec{J}_f$

**Boundary Conditions:**  $\Delta V = 0$ ,  $\Delta \vec{E} = \frac{\sigma}{\epsilon_0} \hat{n} = -\Delta \left( \frac{\partial V}{\partial n} \right) \hat{n}$ ,  $\Delta D_\perp = \sigma_f$ ,  $\Delta \vec{D}_\parallel = \Delta \vec{P}_\parallel$   
 $\Delta \vec{A} = 0$ ,  $\Delta \vec{B} = \mu_0 (\vec{K} \times \hat{n})$ ,  $\Delta \frac{\partial \vec{A}}{\partial n} = -\mu_0 \vec{K}$ ,  $\Delta \vec{H}_\parallel = \vec{K}_{free} \times \hat{n}$ ,  $\Delta H_\perp = -\Delta M_\perp$

**Lorentz Force:**  $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$ , On Wire:  $\vec{F}_{mag} = \int I(d\vec{l} \times \vec{B})$

**Work and Energy:**  $W = Q\Delta V$ ,  $W_{electrostatic} = \frac{\epsilon_0}{2} \int E^2 d\tau$ ,  $W_{elec+polarization} = \frac{1}{2} \int \vec{E} \cdot \vec{D} d\tau$